Suppose S_{k-1} is LI and S_k is LD.

$\because S_k$ is LD then $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_{k-1} v_{k-1} + \alpha_k v_k = 0$ ——— (1)

If $\alpha_k = 0$, then S_k is LI which is a contradiction.

$\Rightarrow \alpha_k \neq 0$.

$$eq(1) \Rightarrow v_k = -\frac{\alpha_1}{\alpha_k} v_1 - \frac{\alpha_2}{\alpha_k} v_2 - \dots\dots - \frac{\alpha_{k-1}}{\alpha_k} v_{k-1}$$

$$\Rightarrow v_k \in [v_1, v_2, \dots\dots, v_{k-1}] \quad , 2 \leq k \leq n$$

which proves the N.P. □

* An infinite subset S of a vector space V is said to LI if every finite subset of S is LI.

Ex:- $\{1, x, x^2, \dots, x^n, \dots\}$

Q: HW

$V = \mathbb{R}^3$

$W_1 = \{(x,y,z) \in \mathbb{R}^3, \; x+y=0\}$

$W_2 = \{(x,y,z) \in \mathbb{R}^3, \; 2x+z=0\}$

Check that W_1, W_2 and $W_1 \cap W_2$ are subspaces or not.