* 48)(b) No. of free variables = dimension

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Theorem:-

If U and W are two ~~finite~~ subspaces of a finite vector space V, then

$$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W).$$

Proof:- Let $\dim(V) = n$

$\dim(U) = m$

$\dim(W) = p$

& $\dim(U \cap W) = r$.

$\therefore\ m \leq n,\ p \leq n,\ r \leq n.$

Let $S_1 = \{v_1, v_2, \ldots, v_r\}$ be the basis of $U \cap W$.

To get the basis of U, we can extend S_1 ~~i.e.~~ as

$S_2 = \{v_1, v_2, \ldots, v_r, u_{r+1}, u_{r+2}, \ldots, u_m\}$

Similarly we can get the basis of W.

$S_3 = \{v_1, v_2, \ldots, v_r, w_{r+1}, w_{r+2}, \ldots, w_p\}$

Now, we have to prove that the set,

$B = \{v_1, v_2, \ldots, v_r, u_{r+1}, \ldots, u_m, w_{r+1}, \ldots, w_p\}$ is a basis for U+W.

$$\sum_{i=1}^{r} \alpha_i v_i + \sum_{i=r+1}^{m} \beta_i u_i + \sum_{i=r+1}^{p} \gamma_i w_i = 0$$

$$\Rightarrow \sum_{i=1}^{r} \alpha_i v_i + \sum_{i=r+1}^{m} \beta_i u_i = -\sum_{i=r+1}^{p} \gamma_i w_i \quad \text{—— (A)}$$