Ex:- $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

① $T(x,y,z) = (x+z,\ z+y)$ (Linear)

② $T(x,y,z) = (x+1,\ y+z)$ (Not linear)

③ $T(x,y,z) = (xy,\ z)$ (Non-linear)

④ $T(x,y,z) = (x+y+z,\ 1)$ (Non-linear)

Identify which is linear?

Sol:- 2) $u_1 = (x_1, y_1, z_1)$

$u_2 = (x_2, y_2, z_2)$

$$T(\alpha u_1 + \beta u_2) = T\left(\alpha(x_1,y_1,z_1) + \beta(x_2,y_2,z_2)\right)$$

$$= T(\alpha x_1 + \beta x_2,\ \alpha y_1 + \beta y_2,\ \alpha z_1 + \beta z_2)$$

$$= (\alpha x_1 + \beta x_2 + 1,\ \alpha y_1 + \beta y_2 + \alpha z_1 + \beta z_2)$$

$$\alpha T(u_1) + \beta T(u_2) = \alpha(x_1+1,\ y_1+z_1) + \beta(x_2+1,\ y_2+z_2)$$

$$= (\alpha x_1 + \alpha,\ \alpha y_1 + \alpha z_1) + (\beta x_2 + \beta,\ \beta y_2 + \beta z_2)$$

$$= (\alpha x_1 + \beta x_2 + \alpha + \beta,\ \alpha y_1 + \alpha z_1 + \beta y_2 + \beta z_2)$$

$\therefore T(\alpha u_1 + \beta u_2) \neq \alpha T(u_1) + \beta T(u_2)$